The Quadratic Formula

Quadratic equations have just one unknown, but contain a square term as well as linear terms.
For example, $2x^2 + x = 3$ is a quadratic equation in x
$7t = 5t^2 + 1$ is a quadratic equation in t.

There is a formula for finding the unknown value, but before it can be used the equation must be written with all of its terms at one side of the equation i.e. in the form $ax^2 + bx + c = 0$ where a, b and c are known positive or negative numbers and x is the unknown value.

The Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$ are: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This formula gives two possible values for x. Usually in practical situations it will be obvious which answer is the correct one, but in some contexts both answers give possible solutions.

Example 1 Solve the equation $2x^2 + x = 3$

How to do it…

Rearrange the equation so all terms are at one side:
In this case subtract 3: $2x^2 + x - 3 = 0$

Write down the values of *a*, *b* and *c*: $a = 2$, $b = 1$ and $c = -3$

Substitute these values into the formula: $x = \dfrac{-1 \pm \sqrt{1^2 - 4\times 2\times -3}}{2\times 2}$

Work out the values in the square root and denominator first: $x = \dfrac{-1 \pm \sqrt{25}}{4}$

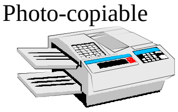
Take the square root (note it is not always a round number)**:** $x = \dfrac{-1 \pm 5}{4}$

Split the formula into two, using + in one and – in the other: $x = \dfrac{-1+5}{4}$ or $x = \dfrac{-1-5}{4}$

Work out the answers: $x = \dfrac{4}{4} = 1$ or $x = \dfrac{-6}{4} = -1.5$

The solutions are x = 1 and – 1.5

Check each answer in the original equation:
Substituting $x = 1$ into $2x^2 + x$ gives $2\times 1^2 + 1 = 3$ Correct!
Substituting $x = -1.5$ into $2x^2 + x$ gives $2\times(-1.5)^2 - 1.5 = 3$ Correct!



Example 2 Solve the equation $7t = 5t^2 + 1$

How to do it…

The working is usually easier if the squared term is positive.

Write the equation the other way round: $5t^2 + 1 = 7t$

Subtract $7t$: $5t^2 + 1 - 7t = 0$

Take care when writing down the values of *a*, *b* and *c*: $a = 5$, $b = -7$ and $c = 1$

$- -7 = 7$

Substitute these values into the formula: $t = \dfrac{7 \pm \sqrt{(-7)^2 - 4 \times 5 \times 1}}{2 \times 5}$

Work out the values in the square root and denominator: $t = \dfrac{7 \pm \sqrt{29}}{10}$

When the square root is not exact, round it to about 4 figures: $t = \dfrac{7 \pm 5.385}{10}$

Split the formula into two: $t = \dfrac{7 + 5.385}{10}$ or $t = \dfrac{7 - 5.385}{10}$

Work out the answers: $t = \dfrac{12.385}{10}$ or $t = \dfrac{1.615}{10}$

The solutions are *t* = 1.24 and 0.16 (to 2dp)

Check each answer in the original equation:

Substituting $t = 1.24$ into each side: LHS: $7t = 7 \times 1.24 = 8.68$

RHS $5t^2 + 1 = 5 \times 1.24^2 + 1 = 8.688$

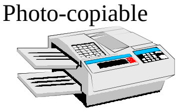
Near enough! The slight discrepancies are due to rounding.

Substituting $t = 0.162$ into each side: LHS: $7t = 7 \times 0.16 = 1.12$

RHS $5t^2 + 1 = 5 \times 0.16^2 + 1 = 1.128$

Solve these:

1 $x^2 + 2x = 8$	2 $3 + 7x + 2x^2 = 0$	3 $22t = t^2 + 21$
4 $t^2 - 7 = 6t$	5 $5x^2 = 2 - 9x$	6 $6p^2 + 5 = 17p$
7 $y^2 - 2y = 4$	8 $2r^2 + 6r = 3$	9 $3q = 2q^2 - 7$
10 $20 = 3r^2 + 5r$	11 $5 - 2x = x^2$	12 $3t^2 = 14t - 5$



Teacher Notes

Unit Intermediate Level, Using algebra, functions and graphs

Notes

This resource includes two examples showing how the quadratic formula can be used to solve quadratic equations together with a set of similar questions for practice. A separate Nuffield resource called ‘Tunnel’ gives examples of quadratic equations in real contexts that can be used when students have become confident with the method. It also shows how problems involving quadratic equations can be solved using graphs in Excel.

Answers (to 2 dp when not exact)

1	2, – 4	2	– 0.5, – 3	3	21, 1	4	7, – 1
5	0.2, – 2	6	2.5, $\frac{1}{3}$	7	3.24, – 1.24	8	0.44, – 3.44
9	2.77, – 1.27	10	1.88, – 3.55	11	1.45, – 3.45	12	4.28, 0.39

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